Afg. 3 Lösungen

(a) $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x - \frac{4}{3}$ ggg.: $(1|0) \in K_f$?

$$\left(\tfrac{1}{3}x^3 - 2x^2 + 3x - \tfrac{4}{3}\right) = (x-1)\left(\tfrac{1}{3}x^2 - \tfrac{5}{3}x + \tfrac{4}{3}\right)$$

$$\begin{array}{l} -\left(\tfrac{1}{3}x^3 - \tfrac{1}{3}x^2\right) \\ \hline -\tfrac{5}{3}x^2 + 3x \\ -\left(-\tfrac{5}{3}x^2 + \tfrac{5}{3}x\right) \\ \hline \tfrac{4}{3}x - \tfrac{4}{3} \\ -\left(\tfrac{4}{3}x - \tfrac{4}{3}\right) \\ \hline 0 \end{array}$$

$$x_{2/3} = \frac{\frac{5}{3} \pm \sqrt{\frac{25}{9} - 4 \cdot \frac{1}{3} \cdot \frac{4}{3}}}{2/3} = \frac{\frac{5}{3} \pm 1}{2/3} \Rightarrow \begin{array}{l} x_2 = 4 \\ x_3 = 1 \end{array}$$

S_x: $\boxed{S_1(1|0)}$ 2-fache NS, $\boxed{S_2(4|0)}$

Ex: $f'(x) = x^2 - 4x + 3$ $f'(x) = 0$ notw. Bed.

$$x_{1/2} = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2} = \frac{4 \pm 2}{2} \Rightarrow \begin{array}{l} x_1 = 3 \\ x_2 = 1 \end{array}$$

$f''(x) = 2x - 4$

$x_1 = 3$ $f''(3) = 2 > 0$ $\boxed{T(3|-\frac{4}{3})}$

hinr. Bed.

$x_2 = 1$ $f''(1) = -2 < 0$ $\boxed{H(1|0)}$

Extrema

Wendepunkt:

$f''(x) = 2x - 4$

$f''(x) = 0$ notw. Bed.

$2x - 4 = 0 \Rightarrow x = 2$

$f'''(x) = 2 \neq 0$ hinr. Bed.

$\boxed{W(2|-\frac{2}{3})}$

(b) $f(x) = x^3 + 3x^2 - 4$ ggg.: $(1|0) \in K_f$?

$$(x^3 + 3x^2 - 4) = (x-1)(x^2 + 4x + 4)$$

$$\begin{array}{l} -(x^3 - x^2) \\ \hline 4x^2 \\ -(4x^2 - 4x) \\ \hline 4x - 4 \\ -(4x - 4) \\ \hline 0 \end{array}$$

$$x_{2/3} = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 4}}{2} = \frac{-4}{2} = -2$$

S_x: $\boxed{S_1(1|0)}$, $\boxed{S_2(-2|0)}$ 2-fache NS

Ex.: $f'(x) = 3x^2 + 6x$ $f'(x) = 0$ notw. Bed.

$3x^2 + 6x = 0 \Leftrightarrow x(6 + 3x) = 0 \Rightarrow x_1 = 0,\ x_2 = -2$

$f''(x) = 6x + 6$

$x_1 = 0$ $f''(0) = 6 > 0$ $\boxed{T(0|-4)}$

hinr. Bedingung

$x_2 = -2$ $f''(-2) = -6 < 0$ $\boxed{H(-2|0)}$

Wendepunkt:

$f''(x) = 0$ n.B. $6x + 6 = 0 \Rightarrow x = -1$

$f'''(x) = 6 \neq 0$ hinr. B.

$\boxed{W(-1|-2)}$

c) $f(x) = x^4 - 5x^2 + 4$

S_x : Subst. $u = x^2$

$u^2 - 5u + 4 = f(u) \qquad u_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 4}}{2} = \frac{5 \pm 3}{2} \Rightarrow \begin{matrix} u_1 = 4 \\ u_2 = 1 \end{matrix}$

Resubst. : $x_{1,2}^2 = 4 \Rightarrow \begin{matrix} x_1 = 2 \\ x_2 = -2 \end{matrix} \qquad x_{3,4}^2 = 1 \Rightarrow \begin{matrix} x_3 = 1 \\ x_4 = -1 \end{matrix}$

$S_1(2|0)$, $S_2(-2|0)$, $S_3(1|0)$, $S_4(-1|0)$

Ex. : $f'(x) = 4x^3 - 10x \qquad f'(x) = 0$ n. Bed.

$4x^3 - 10x = 0$

$\Leftrightarrow x(4x^2 - 10) = 0$

$\Rightarrow x_1 = 0 \quad x_2 = \sqrt{\frac{5}{2}} \quad x_3 = -\sqrt{\frac{5}{2}}$

$f''(x) = 12x^2 - 10$

$x_1 = 0 \quad f''(0) = -10 < 0 \xrightarrow{\text{h. B.}} H(0|4)$

$x_2 = \sqrt{\frac{5}{2}} \quad f''(\sqrt{\frac{5}{2}}) = 20 > 0 \longrightarrow T_1(\sqrt{\frac{5}{2}} | -2{,}25)$ $\left(= -\frac{9}{4}\right)$

Wg. Symmetrie folgt $T_2(-\sqrt{\frac{5}{2}} | -2{,}25)$

W : $f''(x) = 12x^2 - 10 \qquad f''(x) = 0$ n. B.

$12x^2 - 10 = 0$

$\Leftrightarrow 12x^2 = 10$

$x = \pm\sqrt{\frac{5}{6}}$

$f'''(x) = 24x \qquad f'''(\pm\sqrt{\frac{5}{6}}) \neq 0$ h. Bed.

$W_1(\sqrt{\frac{5}{6}} | \frac{19}{36})$, $W_2(-\sqrt{\frac{5}{6}} | \frac{19}{36})$

$f(\sqrt{\frac{5}{6}}) = \frac{25}{36} - 5 \cdot \frac{5}{6} + 4 = -\frac{125}{36} + \frac{144}{36} = \frac{19}{36}$