Klausur 12.1 14/10/09

Vorschläge - Lösungen

Afg. 1: Abl. zur Quotientenregel

(a) $f(x) = \frac{\frac{1}{2}x^2 + 2x - 4}{(x-2)^2}$

$$f'(x) = \frac{(x+2)(x-2)^1 - (\frac{1}{2}x^2 + 2x - 4) \cdot 2(x-2) \cdot 1}{(x-2)^4}$$

$$= \frac{(x^2-4) - (x^2 + 4x - 8)}{(x-2)^3} = \boxed{-\frac{4x-4}{(x-2)^3}}$$

(b) $f(x) = \frac{ax^3 - bx^2 + 4c^2}{1/3\,x}$

$$f'(x) = \frac{(3ax^2 - 2bx)\frac{1}{3}x - (ax^3 - bx^2 + 4c^2) \cdot \frac{1}{3}}{\frac{1}{3} \cdot \frac{1}{3}x^2}$$

$$= \frac{3ax^3 - 2bx^2 - ax^3 + bx^2 - 4c^2}{1/3\,x^2} = \boxed{\frac{2ax^3 - bx^2 - 4c^2}{1/3\,x^2}}$$

(c) $f(x) = \frac{\sin(2x)}{2x^2}$

$$f'(x) = \frac{(\cos(2x) \cdot 2) \cdot 2x^2 - (\sin(2x) \cdot 4x)}{4x^4}$$

$$= \boxed{\frac{x\cos(2x) - \sin(2x)}{x^3}}$$

(d) $f(x) = \frac{x^5 + x^3 + 1}{2x^2 - 4}$

$$f'(x) = \frac{(5x^4 + 3x^2)(2x^2 - 4) - (x^5 + x^3 + 1) \cdot 4x}{(2x^2-4)^2}$$

$$= \frac{10x^6 + 6x^4 - 20x^4 - 12x^2 - 4x^6 - 4x^4 - 4x}{(2x^2-4)^2}$$

$$= \boxed{\frac{6x^6 - 18x^4 - 12x^2 - 4x}{(2x^2-4)^2}}$$

(e) $g(x) = \frac{x^2 + x^2 + 1}{x^2 - 1}$ $\quad g'(x) = \frac{(3x^2+2x)(x^2-1) - 2x(x^3+x+1)}{(x^2-1)^2}$

$= \frac{3x^4 - 3x^2 + 2x^3 - 2x - 2x^4 - 2x^2 - 2x}{(x^2-1)^2}$ $\quad = \frac{x^4 - 3x^2 - 4x}{(x^2-1)^2}$