Lösung A — Mathe Klausur 12-2, 07.04.2008 — Lösung B

A1 (Lösung A)

$g(x) = \cos\left(\frac{x}{x^2+1}\right)$

2P $g'(x) = \underbrace{-\sin\left(\frac{x}{x^2+1}\right)}_{1P} \cdot \underbrace{\frac{x^2+1-2x^2}{(x^2+1)^2}}_{1P}$

A1 (Lösung B)

$g(x) = \sin\left(\frac{x}{x^2+1}\right)$

$g'(x) = \underbrace{\cos\left(\frac{x}{x^2+1}\right)}_{1P} \cdot \underbrace{\frac{-x^2+1}{(x^2+1)^2}}_{1P}$ 2P

2a) (Lösung A)

$h(x) = -\frac{\pi^2}{3}\cdot\cos\left(\frac{3}{\pi}(x-k^2)\right)$

2P $H(x) = \underbrace{-\frac{\pi^3}{9}}_{1P}\cdot\underbrace{\sin\left(\frac{3}{\pi}(x-k^2)\right)}_{1P}$

2a (Lösung B)

$h(x) = -\frac{\pi^2}{3}\cos\left(\frac{\pi}{3}(x-k^2)\right)$

$H(x) = \underbrace{-\pi}_{1P}\underbrace{\sin\left(\frac{\pi}{3}(x-k^2)\right)}_{1P}$ 2P

2b) (Lösung A) 4P

$\int_0^k 2\cdot\sin\left(\frac{1}{2}x\right)dx = \left[-4\cos\left(\frac{1}{2}x\right)\right]_0^k$ 1 1

$-4\cos\frac{k}{2} - (-4)\cdot 1 = 4$ 1

$-4\cos\frac{k}{2} = 0 \Rightarrow \frac{k}{2} = \frac{\pi}{2}$ $\underline{k=\pi}$ 1

2b (Lösung B) 4P

$\int_0^k \sin\left(\frac{1}{2}x\right)dx = \left[-2\cos\left(\frac{1}{2}x\right)\right]_0^k$ 1 1

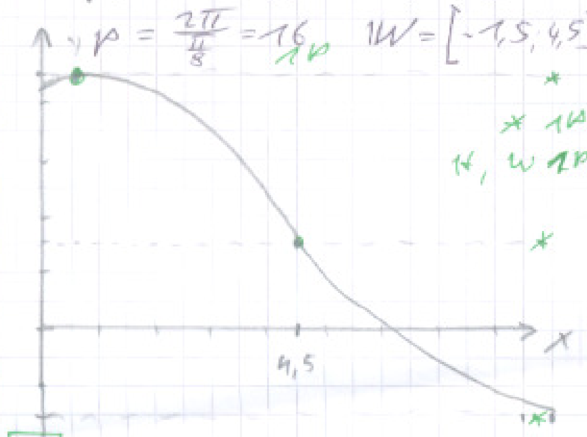
$-2\cos\frac{k}{2} - (-2)\cdot 1 = 4$ 1

$-2\cos\frac{k}{2} = 2 \quad \cos\frac{k}{2} = -1$ $\underline{k=2\pi}$ 1

3a) (Lösung A)

$f(x) = 3\cos\left(\frac{\pi}{8}\left(x-\frac{1}{2}\right)\right) + \frac{3}{2}$

$p = \frac{2\pi}{\frac{\pi}{8}} = 16$ 1P $\quad W = [-1{,}5;\ 4{,}5]$



* 1P

H, W 1P

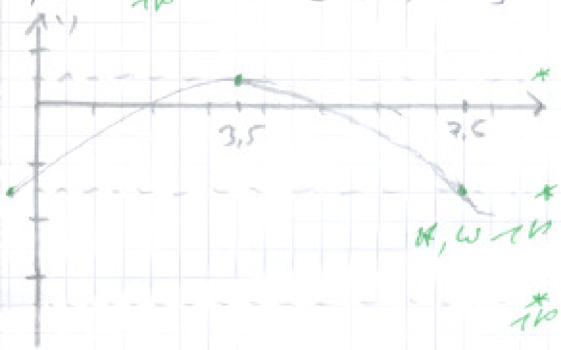
Schaubild 1P

4P

3a (Lösung B)

$f(x) = 2\sin\left(\frac{\pi}{8}\left(x+\frac{1}{2}\right)\right) - \frac{3}{2}$

$p = 16$ 1P $\quad W = [-3{,}5;\ 0{,}5]$



H, W 1P

* 1P

Schaubild 1P

4P

3b) (Lösung A) 4P

$p = \frac{3}{4}\pi - \left(-\frac{5}{4}\pi\right) = 2\pi$ 1 $\quad d=0,\ a=4$ 1 $\quad b=1,\ c=\frac{\pi}{4}$ 1

$\underline{y = 4\cdot\sin\left(x+\frac{\pi}{4}\right)}$ 1

3b (Lösung B) 4P

$p = 2\pi$ 1 $\quad d=0,\ a=2$ 1 $\quad b=1,\ c=-\frac{\pi}{4}$ 1

$\underline{y = 2\sin\left(x-\frac{\pi}{4}\right)}$ 1

A4 (Lösung A) 4P

$V = \pi\int_0^{\pi}[\sin x]^2\,dx$ 1

$V = \pi\cdot\frac{\pi}{6}\left[1\cdot 0^2 + 4\cdot 1^2 + 1\cdot 0^2\right]$ 2

$V = \frac{2}{3}\pi^2$ 1

$\boxed{\text{oder}}$

A4 (Lösung B) 4P

$V = \pi\int_0^{\pi}[\sin x]^2\,dx$ 1

$f(0) = 0 \quad f\left(\frac{\pi}{2}\right) = 1 \quad f(\pi) = 0$ 2

$V = \pi\cdot\frac{\pi-0}{6}(0+4+0) = \frac{2}{3}\pi^2$ 1