Martin Wellmann Aufgabe 6 10.1.2009

$f_t(x) = 4e^{tx} - e^{2tx} = e^{tx}\cdot(4 - e^{tx})\ ;\ t>0$

a) $f_t'(x) = 4te^{tx} - 2te^{2tx} = 2te^{tx}\cdot(2 - e^{tx})$

$f_t''(x) = 4t^2e^{tx} - 4t^2e^{2tx} = 4t^2e^{tx}(1 - e^{tx})$

$f_t'''(x) = 4t^3e^{tx} - 8t^3e^{2tx} = 4t^3e^{tx}(1 - 2e^{tx})$

b) $f_t(0) = 4e^0 - e^0 = 4 - 1 = 3$ $\quad S_y(0/3)$

$f_t(x) = 0 \qquad e^{tx} \neq 0 \Rightarrow 4 - e^{tx} = 0$

$4 = e^{tx} \qquad x = \frac{\ln 4}{t} \qquad N_t\left(\frac{\ln 4}{t}/0\right)$

$f_t'(x) = 0 \qquad 2 = e^{tx} \qquad x = \frac{\ln 2}{t}$

$f_t''(x) = 4t^2 \cdot e^{\ln 2}\cdot(1 - e^{\ln 2}) = 4t^2 \cdot 2(1-2)$

$= -8t^2 < 0 \qquad f_t\left(\frac{\ln 2}{t}\right) = 2\cdot(4-2) = 4$

$\boxed{\text{zuh Ortslinie der HP: } y = 4}$ $\quad H_t\left(\frac{\ln 2}{t}/4\right)$

$f_t''(x) = 0 \qquad 1 = e^{tx} \qquad x = \frac{\ln 1}{t} = 0$

$f_t'''(0) = 4t^3\cdot(1-2) = -4t^3 \neq 0 \qquad W = S_y(0/3)$

c) $e^{tx} \xrightarrow[x\to\infty]{} \infty \Rightarrow f_t(x) \xrightarrow[x\to\infty]{} \text{„}\infty\cdot(-\infty)\text{“} = -\infty$

$e^{tx} \xrightarrow[x\to-\infty]{} 0 \Rightarrow f_t(x) \xrightarrow[x\to-\infty]{} 0\cdot 4 = 0$

$\Rightarrow$ Die negative x-Achse ist Asymptote

d) + e) + f) + g)

x	$f_1(x)$	$f_{0,25}(x)$	$f_{0,5}(x)$
-6	0,010	0,843	0,197
-5	0,027		
-4	0,073	1,336	0,523
-2	0,523	2,058	1,336
0	3	3	3
0,693	4		
2,773		4	
1,386			4
1,386	0		
5,545		0	
2,773			0

$\boxed{e \rightarrow}$ $\frac{y-3}{x} = m \qquad f_t'(0) = 2t$

$t: y = 2tx + 3 \qquad n: y = -\frac{1}{2t}x + 3$

$y=0 \quad A_t\left(-\frac{3}{2t}/0\right) \qquad y=0 \quad B_t(6t/0)$

$A(t) = \frac{1}{2}\left(6t - \left(-\frac{3}{2t}\right)\right)\cdot 3$

GTR: A ist für $t = \frac{1}{2}$ min.

$A_{\frac{1}{2}}(-3/0) \quad B_t(3/0)$

$\boxed{g \rightarrow}$ Δ_{ABS_y} ist symm. zur y-Achse

$\Rightarrow$ Es ist gleichsch. u. rechtw.