Lösungsblatt 9 erstellt von Frau Zepf

Flächen unterhalb der x - Achse : Lösungen

(1) (a) $f(x) = \frac{1}{4}x^4 - 4$

NS : Substitution $x^2 = u$ $\quad f(u) = \frac{1}{4}u^2 - 4 = 0$

$u_{1/2} = \frac{\pm\sqrt{-4\cdot\frac{1}{4}\cdot(-4)}}{\frac{1}{2}} = \pm\frac{2}{\frac{1}{2}} \Rightarrow u_1 = 4$

$u_2 = -4$ ↯

Resubstitution : $x^2 = 4 \Rightarrow x_{1/2} = \pm 2$

A : $-\int_{-2}^{2} f(x)\,dx = -\int_{-2}^{2}\left(\frac{1}{4}x^4 - 4\right)dx = -\left[\frac{1}{20}x^5 - 4x\right]_{-2}^{2}$

$= -\left[F(2) - F(-2)\right] = -(-6{,}4 - 6{,}4) = 12{,}8$

(b) $f(x) = x^4 - 4x^2$

NS : $x^2(x^2-4) = 0 \quad x_1 = 0 \quad x_{2/3} = \pm 2$

A : $-\int_{-2}^{2} f(x)\,dx = -\int_{-2}^{2}(x^4 - 4x^2)\,dx = -\left[\frac{1}{5}x^5 - \frac{4}{3}x^3\right]_{-2}^{2}$

$= -\left[F(2) - F(-2)\right] = -(-4{,}2\overline{6} - 4{,}2\overline{6})$

$= 8{,}5\overline{3}$

(c) $f(x) = x^3 - 6x^2 + 9x - 4$

NS : Polynomdivision $x_1 = 1$ $\quad (x^3 - 6x^2 + 9x - 4) = (x-1)(x^2 - 5x + 4)$

$-(x^3 - x^2)$

$-5x^2 + 9x$

$-(-5x^2 + 5x)$

$4x - 4$

$-(4x - 4)$

0

$x_{2/3} = \frac{5 \pm \sqrt{25 - 4\cdot1\cdot4}}{2} = \frac{5 \pm 3}{2}$

$\Rightarrow x_2 = 4 \quad x_3 = x_1 = 1$

A : $-\int_{1}^{4} f(x)\,dx = -\int_{1}^{4}(x^3 - 6x^2 + 9x - 4)\,dx = -\left[\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 - 4x\right]_{1}^{4}$

$= -\left[F(4) - F(1)\right] = -\left[-8 - (-1{,}25)\right] = 6{,}75$

(d) $f(x) = x^2 + 6x + 8$

NS : $x_{1/2} = \frac{-6 \pm \sqrt{36 - 4\cdot1\cdot8}}{2} = \frac{-6 \pm 2}{2} \Rightarrow x_1 = -4$, $x_2 = -2$

A : $-\int_{-4}^{-2} f(x)\,dx = -\left[\frac{1}{3}x^3 + 3x^2 + 8x\right]_{-4}^{-2}$

$= -\left[F(-2) - F(-4)\right] = \frac{4}{3}$

(2) (a) $\int_0^b x^2\,dx = 9$

$\int_0^b x^2\,dx = \left[\frac{1}{3}x^3\right]_0^b$

$= \frac{1}{3}b^3 - 0 = 9$

$\Rightarrow b^3 = 27 \Rightarrow b = 3$